

Three-dimensional modelling of cohesive sediment transport in the Loire estuary

C. Le Normant*

Laboratoire National d'Hydraulique et Environnement, EDF, 6 quai Watier, 78 400 Chatou, France

Abstract:

In the framework of a European research programme, processes describing cohesive sediment transport, including fluid mud movements and consolidation, have been modelled and implemented in the three-dimensional finite element model TELEMAC-3D, which solves free surface flow problems. The model has been applied to several European estuaries and among them, the Loire estuary in France. This application proved the ability of the code to reproduce typical estuarine features such as the formation of a turbidity maximum and the movements of fluid mud. Copyright © 2000 John Wiley & Sons, Ltd.

KEY WORDS sediment transport; estuarine processes; cohesive sediment

INTRODUCTION

Estuaries, the confluence zones between river and sea, show a complex dynamic behaviour. Saltwater intrusion can lead to a more or less well-defined salt wedge with associated density currents. Fine sediment transport in estuaries can lead to the occurrence of a turbidity maximum, which induces problems, both ecological (cohesive sediment absorbs heavy metals and pesticides and reduces light penetration) and economic (as deposition rates are high near the location of the turbidity maximum, expensive dredging is needed). One way to study the sediment dynamics in an estuary is with a numerical model. Three-dimensional models seem particularly appropriate here owing to the complex bathymetries (tidal flats, navigation channel), the vertical gradients in salinity and suspended sediment concentration, density currents, etc. It was thus decided, in the framework of the European project MAST, to extend the LNH three-dimensional finite element model TELEMAC-3D solving free surface flow problems to cohesive sediment transport. The sediment module consists of the suspended sediment transport equation, deposition and erosion, a 2-D depth integrated model of fluid mud movements and a 1-D vertical model of mud bed consolidation (Le Normant *et al.*, 1993). The model was then applied to several European estuaries, in particular the Loire estuary, to prove its ability to reproduce the known processes.

DESCRIPTION OF THE MODEL

The finite element software TELEMAC-3D solves the Navier–Stokes equations with a free surface boundary condition and the advection–diffusion equations for the temperature, the salinity and any other variables (Janin *et al.*, 1992). Density effects, wind stress on the free surface, heat exchange with the atmosphere and the Coriolis force are included in the model. Variations of the density are taken into account in the momentum equations via the Boussinesq approximation. The physical problems considered allow us to

*Correspondence to: Dr C Le Normant, Laboratoire National d'Hydraulique, EDF, 6 quai Watier, 78 400 Chatou, France.
E-mail: Catherine.Le-Normant@edf.fr

assume that the pressure is hydrostatic. The equations are solved by means of a decomposition into fractional steps (an advection step, a diffusion step and a surface-continuity–pressure step). The space discretization is achieved using prisms with quadrilateral sides. The horizontal 2-D projection of the mesh is made of triangles. We therefore need only to mesh the 2-D horizontal domain and then duplicate it along the vertical.

To model sediment transport in estuaries, three specific modules were developed.

The suspended sediment module

The transport of suspended sediment is described by the following advection–diffusion equation

$$\frac{\partial C}{\partial t} + u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} + w \frac{\partial C}{\partial z} + \frac{\partial (W_c C)}{\partial z} = \frac{\partial}{\partial x} \left(K_h \frac{\partial C}{\partial x} \right) + \frac{\partial}{\partial y} \left(K_h \frac{\partial C}{\partial y} \right) + \frac{\partial}{\partial z} \left(K_z \frac{\partial C}{\partial z} \right) \quad (1)$$

where C is the concentration of the suspended sediment, u , v and w are the velocity components, and K_h and K_z are the eddy diffusivity coefficients. To model the density stratification, the vertical eddy diffusivity is described by a mixing length model with damping functions. In this approach, the stability of the stratification is characterised by the gradient Richardson number. The settling velocity W_c is a function of concentration, salinity and temperature to reproduce flocculation. Its expression is empirical and depends on the estuary studied.

The resolution of Equation (1) is achieved in two steps, an advection step and a diffusion step (the term $\partial(W_c C)/\partial z$ is treated in the diffusion step as it physically counterbalances the vertical turbulent flux of concentration, once the boundary conditions are given.

At the surface, the net sediment flux is zero but at the sediment–water interface, the flux is the result of erosion and sedimentation. The bottom boundary condition therefore can be formulated as follows

$$-K_z \frac{\partial C}{\partial z} + W_c C_{\text{Bot.}} = F_{\text{erosion}} + F_{\text{deposition}} \quad (2)$$

The erosion rate is represented by Partheniades' formulation (Partheniades, 1965)

$$F_{\text{erosion}} (\text{kg/m}^2/\text{s}) = M \left(\frac{\tau_b}{\tau_{ce}} - 1 \right) \quad \text{for } \tau_b > \tau_{ce} \quad (3)$$

where τ_b is the bed shear stress and τ_{ce} , the critical shear stress for erosion, is a function of the concentration of the top bed layer, which itself is given by the state of consolidation. The erosion coefficient M may also be a function of the concentration.

This expression of the erosion rate is generally used when simulating sediment transport in the Loire estuary (Le Hir and Thouvenin, 1992). However, it has been established for consolidated mud beds. When dealing with fluid mud, an entrainment formula is more suitable to quantify the sediment entrainment rate that occurs at the fluid mud–water interface. In such a formula, the Richardson number, calculated from the density step and the velocity jump across the interface, is the governing parameter. Srivinas and Mehta (1990) simulated fluid mud entrainment experimentally by current shear and derived an expression for the fluid mud entrainment rate. We are planning, in the future, to implement and test this formula.

The deposition rate is calculated according to Krone's formula (Krone, 1962)

$$F_{\text{deposition}} (\text{kg/m}^2/\text{s}) = P_d W_c C \quad (4)$$

in which P_d is the probability for deposition

$$P_d = 1 - \left(\frac{\tau_b}{\tau_{cd}} \right) \quad \text{for } \tau_b < \tau_{cd} \quad (5)$$

with τ_{cd} the critical shear stress for deposition. According to Odd and Cooper (1989), the critical shear stress

for deposition on the fluid mud layer is expected to be higher than the critical value for deposition on the bed and two distinct values of τ_{cd} should thus be considered. For the application to the Loire estuary, the available field data did not enable us to make such a distinction.

The fluid mud module

Fluid mud is a dense sediment layer located on the bottom. It is formed during low energy conditions, when the mud flocs settle and concentrate near the bed. This layer of 0.5 m–2 m thick and with concentrations between 50 g/l and 200 g/l is able to flow horizontally under the influence of gravity, hydrostatic forces or the overlying water currents. As fluid mud plays an important role in the total fine sediment transport in many macrotidal estuaries (the Loire estuary for instance), modelling its movements presents a real interest.

This dense suspension contains a concentration of mud flocs that is high enough to cause a significant change in the physical properties of the mud–water mixture compared with those of clear water. Fluid mud in different sites can have different rheological behaviours. For the Loire estuary, Migniot (1982) studied the behaviour of muds with different concentrations and he determined that below 250 g/l, the mud flows as a viscous fluid.

To represent the fluid mud movements, a depth integrated model has then been developed where the mud is treated as a Newtonian fluid with a high viscosity (Malcherek *et al.*, 1996).

Several assumptions are made in the model:

1. the mean concentration of the layer, C_m , is assumed to be constant;
2. the effect of the overlying water on the fluid mud movements is only exerted through pressure, shear stress at the interface and mass exchanges at the water–fluid mud interface;
3. the water pressure and the fluid mud pressure are hydrostatic.

The equations solved by the model are then

$$\frac{\partial d_m}{\partial t} + \frac{\partial(d_m \bar{u}_m)}{\partial x} + \frac{\partial(d_m \bar{v}_m)}{\partial y} - \frac{1}{C_m} \frac{dM}{dt} = 0 \quad (6)$$

$$\frac{\partial \bar{u}_m}{\partial t} + \bar{u}_m \frac{\partial \bar{u}_m}{\partial x} + \bar{v}_m \frac{\partial \bar{u}_m}{\partial y} = -\frac{\rho_m - \rho_o}{\rho_m} g \frac{\partial(Zf + d_m)}{\partial x} - \frac{\rho_o}{\rho_m} g \frac{\partial(S)}{\partial x} + D \left(\frac{\partial^2 \bar{u}_m}{\partial x^2} + \frac{\partial^2 \bar{u}_m}{\partial y^2} \right) + \frac{\tau_{ix} - \tau_{ox}}{\rho_m d_m} - 2\Omega \bar{v}_m \quad (7)$$

$$\frac{\partial \bar{v}_m}{\partial t} + \bar{u}_m \frac{\partial \bar{v}_m}{\partial x} + \bar{v}_m \frac{\partial \bar{v}_m}{\partial y} = -\frac{\rho_m - \rho_o}{\rho_m} g \frac{\partial(Zf + d_m)}{\partial y} - \frac{\rho_o}{\rho_m} g \frac{\partial(S)}{\partial y} + D \left(\frac{\partial^2 \bar{v}_m}{\partial x^2} + \frac{\partial^2 \bar{v}_m}{\partial y^2} \right) + \frac{\tau_{iy} - \tau_{oy}}{\rho_m d_m} + 2\Omega \bar{u}_m \quad (8)$$

where d_m is thickness of the fluid mud layer, ρ_m is density of the fluid mud layer, (\bar{u}_m, \bar{v}_m) is fluid mud velocity, ρ_o is water density, Zf is the bottom level, S is the free surface, D is the diffusion coefficient and Ω is the Coriolis parameter.

The expression dM/dt represents the net rate of mass exchange of mud between the fluid mud layer and the water column (deposition from the water column and entrainment of the fluid mud by the water flow) and between the fluid mud and the consolidated bed below (dewatering of fluid mud and erosion of the consolidated bed by the fluid mud flow). The shear stress at the fluid mud–water interface, τ_i , and the shear stress at the fluid mud–consolidated bed interface, τ_o , are expressed as quadratic functions of the velocities.

This model is linked to the 3-D model for the movement of suspended sediment and to the 1-D model for the consolidation.

The consolidation module

During consolidation, the strain initially acting on the pore fluid of the sediment is progressively transmitted to the solid grains and effective stresses develop. The water content of the mud decreases then along with the bed height.

To represent the evolution of the height and concentration of the mud bed, a model based on Gibson's theory (Gibson *et al.*, 1967) was developed. It calculates the evolution of the void ratio e of a soil layer

$$\frac{\partial e}{\partial t} + \left(\frac{\rho_s}{\rho_f} - 1 \right) \frac{d}{de} \left(\frac{k}{1+e} \right) \frac{\partial e}{\partial z_s} + \frac{\partial}{\partial z_s} \left(\frac{1}{\rho_f g} \frac{k}{1+e} \frac{d\sigma'}{de} \frac{\partial e}{\partial z_s} \right) = 0 \quad (9)$$

where ρ_s is the solid density, ρ_f is the fluid density and z_s is a reduced material coordinate deduced from the vertical coordinate z : $\delta z = \delta z_s(1+e)$. The constitutive relationships for the permeability k and the effective stress σ' as functions of the void ratio are needed to close this equation.

To solve this equation, a vertical mesh that discretizes the mud bed into many layers is considered. For computational efficiency, the time step for the consolidation model can be much longer than for the hydrodynamics.

To be introduced in the code, the consolidation model had to be linked to the modelling of the two mechanisms, that govern mass exchanges between the mud bed and the overlying fluid, namely deposition and erosion. In this manner, the deposition affects the top bed layer and the fresh deposits consolidate along with the other bed layers. The erosion flux is different for each layer because the resistance to erosion of a mud layer is related to its concentration.

The Gibson model needs to consider at each point of the 2-D mesh a vertical mesh with many layers, which is time consuming. Moreover it requires the constitutive relationships $k(e)$ and $\sigma'(e)$, which are rarely available. It may then be difficult to apply it when studying real estuaries. In these conditions, a simpler model for bed consolidation was also implemented in TELEMAC-3D. This model was previously developed at the Laboratoire National d'Hydraulique and had been used for real applications. It consists of a multilayer model. In this model, the bed is divided into several layers characterized by their concentration C_s and their residence time T_s . The quantity of mud that remains in the layer M after the time $T_s(M)$ goes into the more consolidated layer $M+1$. The thickness $E(M+1)$ of the latter then increases according to

$$E(M+1) = E(M+1) + E(M) \frac{C_s(M)}{C_s(M+1)} \quad (10)$$

and

$$E(M) = 0 \quad (11)$$

As a consequence, the bottom level decreases.

APPLICATION TO THE LOIRE ESTUARY

The Loire estuary features

The Loire estuary, located on the French Atlantic coast (Figure 1), is a macrotidal estuary with a tidal range at the mouth varying between 2.3 and 5 m and an average flow rate over the year of 800 m³/s. It exhibits the typical features of turbidity maximum and fluid mud of a macrotidal estuary with up to 1 million tons of suspended sediment in the turbidity maximum and fluid mud formation under neap tides.

The results introduced in the present paper were obtained for spring tide conditions. The turbidity maximum is then present during the whole tidal cycle, whereas during neap tides, sediment settles to form fluid mud and the turbidity maximum almost disappears.

Numerical grid

The domain considered for the computations spreads to St-Florent le Viel, located 105 km from the mouth (St Nazaire), upstream from the extreme limit of tidal propagation, which reaches now nearly 100 km for a spring tide and a low river discharge. As the study domain is rather extended (Figure 2), the grid was generated with a view of limiting computing costs. Therefore, larger mesh sizes were generated in the

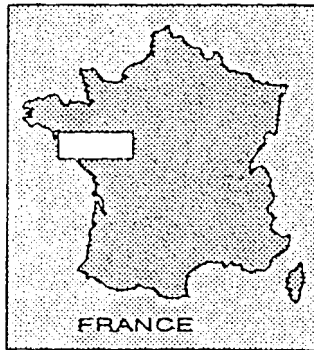


Figure 1. Location of the Loire Estuary

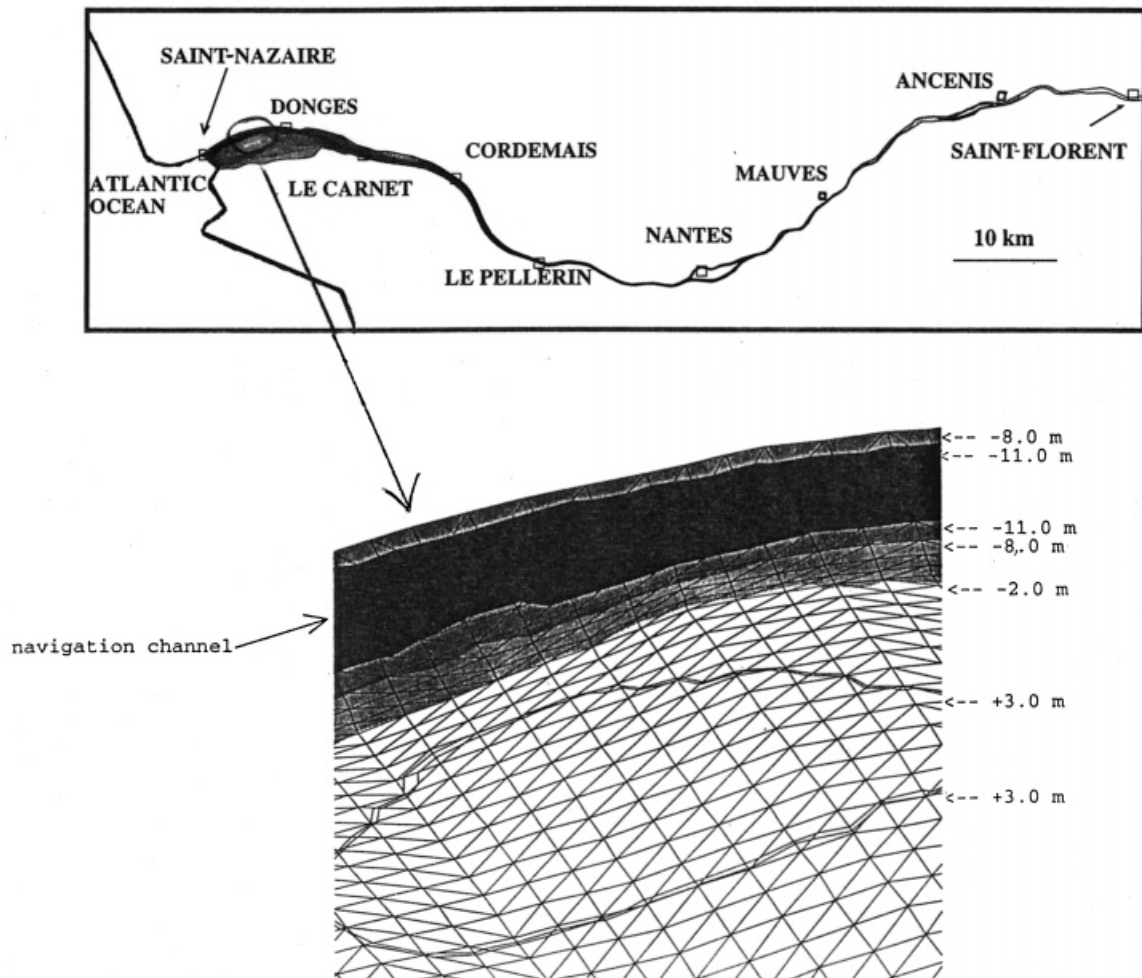


Figure 2. Loire geometry and numerical mesh details

direction of the flow. However, the mesh must be refined in the downstream part of the estuary where the navigation channel induces strong bottom gradients (Figure 2). The total number of nodes for the computation is 40 000 when the hydrodynamic conditions simulated lead to a stratified estuary (the vertical plane is then discretized using 10 levels).

Model parameters

Hydrodynamic parameters. The numerical time step takes into account the size of the smallest meshes and the maximum flow velocity. A 10 s time step was considered for all the simulations.

Eddy viscosities are based on a mixing length model with Munk and Anderson damping functions. As no vertical measurements were available for the period simulated, no other turbulence closure was tried.

Friction coefficients were calibrated to reproduce tide propagation in the estuary. Five areas with different Chezy coefficients were defined. The order of magnitude of the Chezy coefficient is correlated with bottom characteristics (mud downstream and sand upstream).

Numerical simulations show that a Chezy coefficient variation of 20 in the downstream part of the estuary has no real influence on free surface elevation and currents. On the other hand, the same modification between Cordemais (26 km upstream) and Le Pellerin (38 km upstream), where water depth is less important, leads to strong variation in the results.

Sediment parameters. The settling velocity was imposed as a function of the suspended sediment concentration according to the results of laboratory experiments (Migniot, 1982; Sanchez Angulo, 1992). The formula leads to a peak settling velocity of 1 mm/s.

Bottom concentration is assumed to be constant (150 g/l). The consolidation phenomenon has not been modelled because only a few tides were reproduced. When tidal cycles are simulated, however, consolidation should be taken into account because it can modify the characteristics of the turbidity maximum. Indeed, at the end of neap tides, part of the deposits have had time to consolidate and could not be eroded again, depending on the next tidal coefficients.

Based on Migniot's observation, critical shear stress for deposition is 0.7 N/m² and critical shear stress for erosion is 0.4 N/m².

Partheniades erosion parameter is calculated by Cormault's formula, established for the Gironde's estuary.

Numerical simulations

Hydrodynamic results. We first made sure that tidal propagation and salt intrusion in the estuary were reproduced correctly, as these determine sediment movement.

The tidal curve at the mouth was decomposed using a harmonic analysis, and was used as the downstream boundary condition. Daily flow discharge was imposed upstream. Salinity at the mouth was imposed at its measured value during flood and could leave the estuary during ebb.

Numerical simulations were checked by the mean of tidal and salinity curves available in several locations along the estuary. Comparisons of surface elevations and salinities show a good agreement between measured and computed results (Figures 3 and 4). For example, for a spring tide with a low water discharge, the longitudinal computed profile of surface salinities at high water fits well with measurements done for such hydrodynamic conditions (Figure 5).

Sediment results. River water turbidity was imposed upstream. According to measurements carried out between May 1981 and August 1982, it could be estimated using the following formula

$$C(\text{mg/l}) = 40 \log(Q_f) - 80 \quad (12)$$

At the mouth, suspended concentration was imposed during flood. For the simulation of a spring tide with a low water discharge, the concentration of marine water (10 mg/l) was imposed, as the turbidity maximum

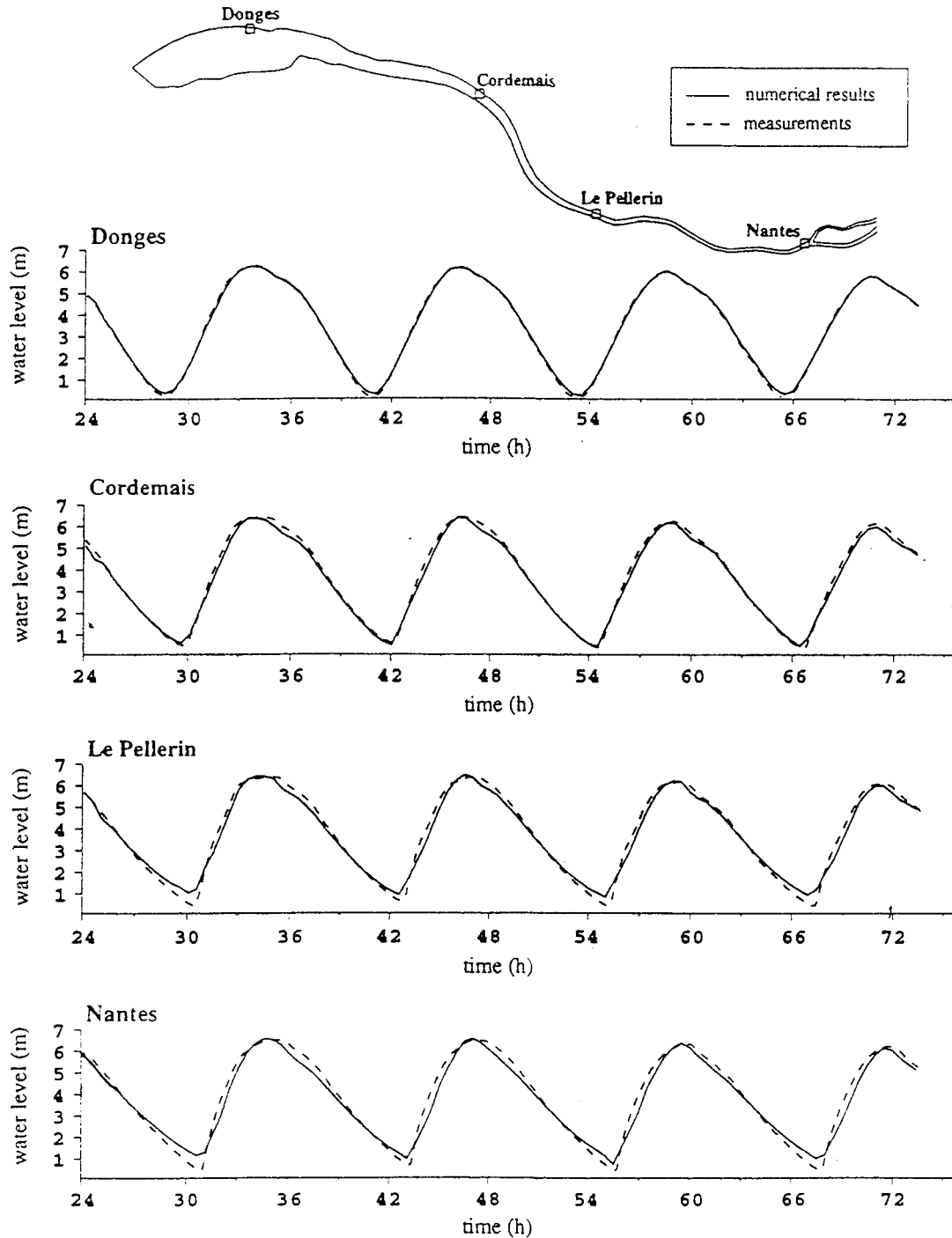


Figure 3. Surface elevations for a spring tide with a low river discharge

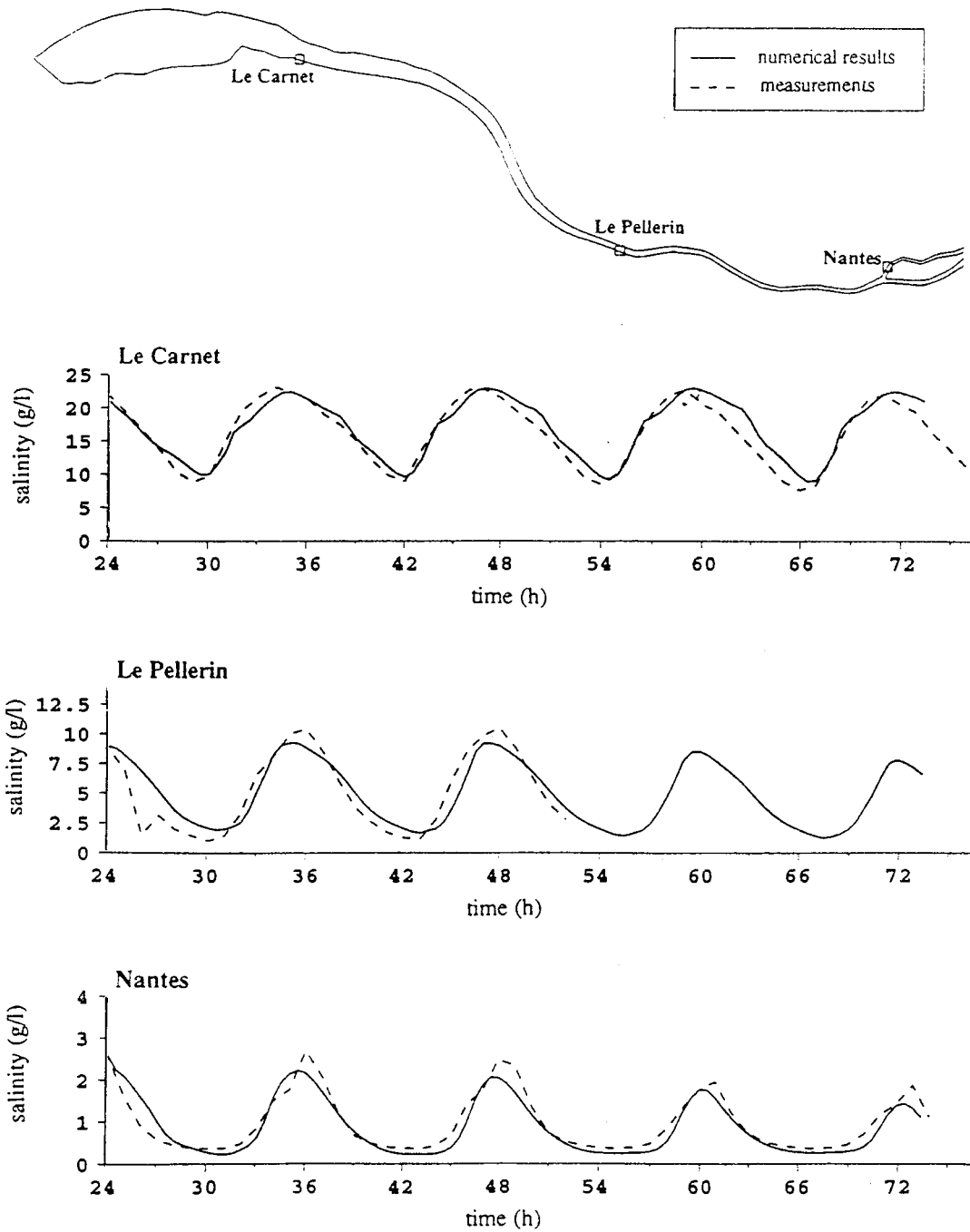


Figure 4. Surface salinities for a spring tide with a low river discharge

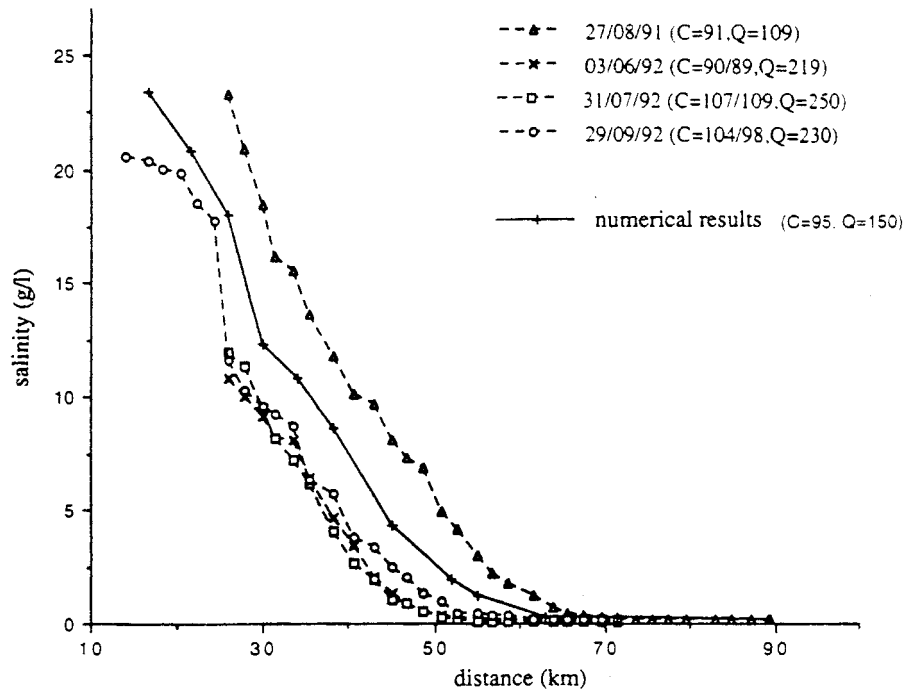


Figure 5. Longitudinal surface salinities profiles

then remains in the estuary. For the simulation of a mean river discharge, measurements of sediment discharge carried out in 1981 and 1982 were used and a concentration of 0.4 g/l was imposed during flood.

To minimize computational costs, SUBIEF, a 2-D suspended transport model, was used to initialize the 3-D runs. As an initial condition, the whole sediment was considered as a uniform deposit located in the downstream part of the estuary. For a spring tide with a low water discharge, its thickness was 28 cm to obtain a sediment volume of about 500 000 tons (value of the suspended mass estimated in 1981 for such hydrodynamic conditions, CSEEL, 1983). Seven tides were simulated. As early as the first tide deposits were eroded, and at the last tide the turbidity maximum was located between Donges (10 km upstream) and Nantes (60 km upstream). Another computation, with different initial conditions, was made to establish that the initial distribution of the deposits did not govern the final result.

The data available to check the model results are scattered in time and space. As the Loire estuary has undergone significant modification over time, owing to dredging and sand extraction, a consistent set of field measurements does not exist for this estuary and the numerical results can be judged only qualitatively. Results obtained for two river flow rates (Figure 6) show that the model reproduces the observed locations and lengths of the turbidity maximum. For a low water discharge, the turbidity maximum is located between 10 (Donges) and 50 km upstream (Nantes). This result is in agreement with the observations achieved in the Loire estuary for such conditions (Figure 7). During the tide, its oscillation is about 48 km, a value which corresponds to that noted by Gallenne (1974): between 40 and 50 km. For a mean water discharge (about 900 m³/s), the turbidity maximum is located further downstream than during a low water discharge. Its upstream limit position is located at 35 km. This is consistent with the observations of Gallenne (1974) who situated the upstream limit of the turbidity maximum between Cordemais (26 km upstream) and La Martinière (37 km upstream) for river flow discharges between 500 m³/s and 1000 m³/s.

At the turbidity maximum simulated by the model, the vertical gradients of concentration can be quite strong, leading to a concentration of 1 g/l at the surface and 20 g/l at the bottom (Figure 6). This result is

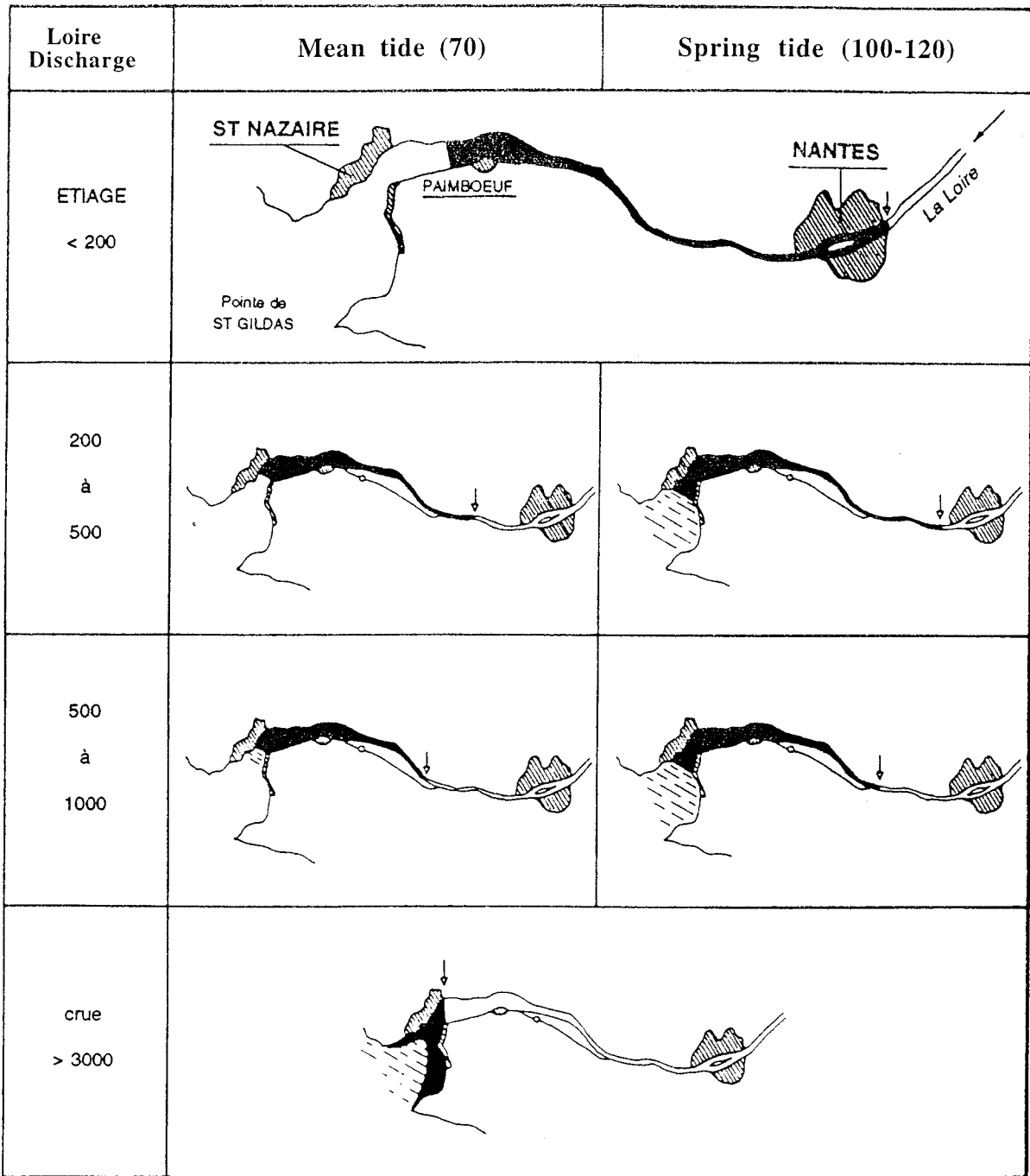


Figure 6. Landward position of the turbidity maximum depending on the river discharge (model results)

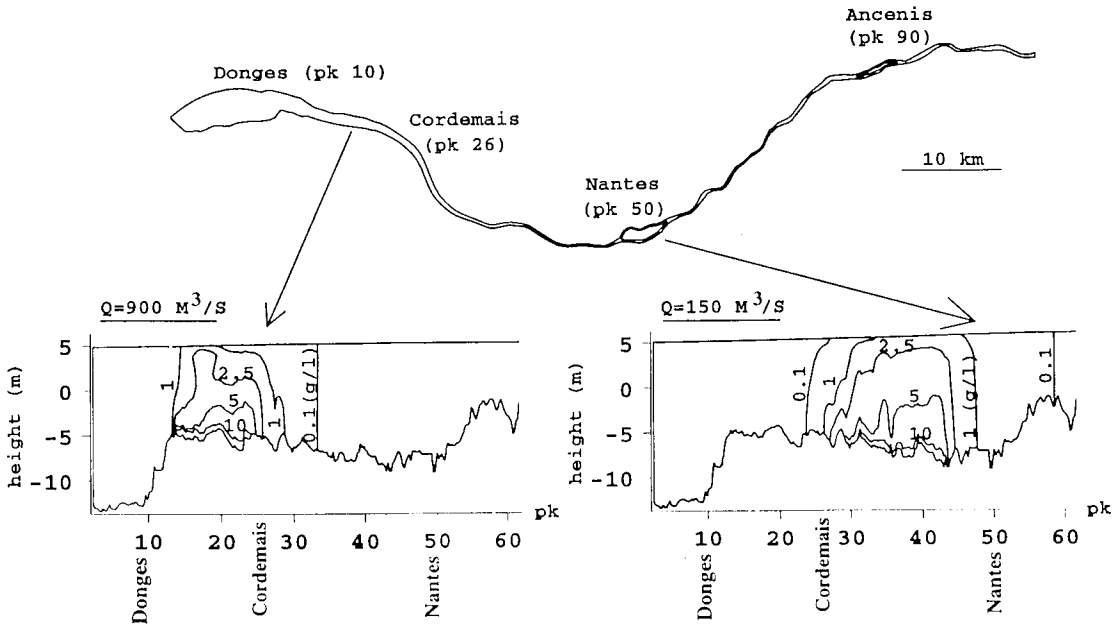


Figure 7. Position of the turbidity maximum depending on the river discharge and the tidal coefficient

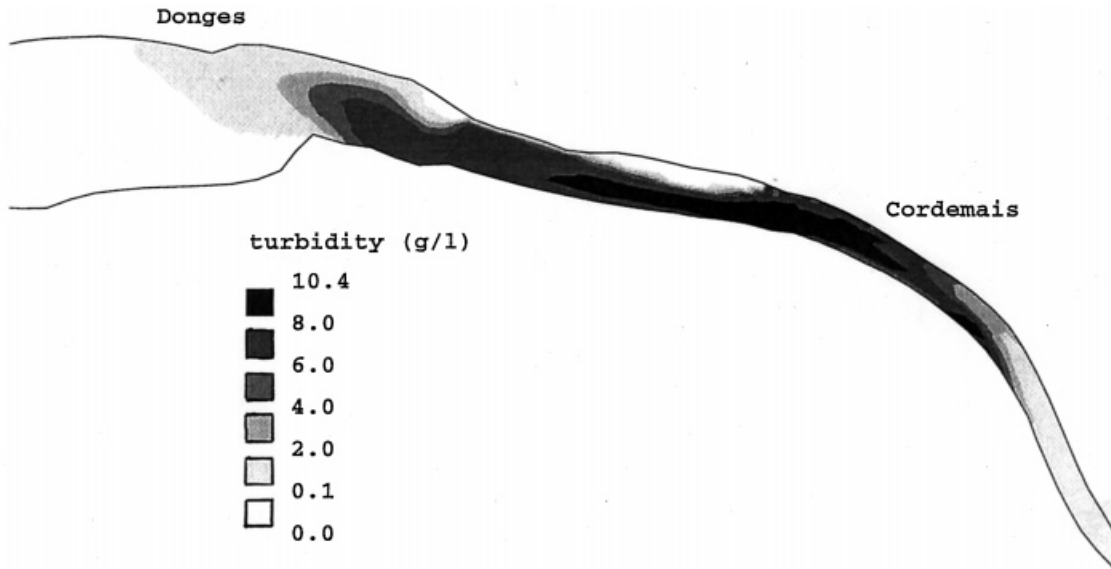


Figure 8. Depth-averaged turbidities at the end of ebb (spring tide and low river discharge)

in agreement with measurements made in the estuary (CSEEL, 1983). The distribution of the depth-averaged concentrations in the estuary at the end of ebb has been established from the model results (Figure 8). It shows that the suspended sediment is transported mainly in the navigation channel, as is observed in the field. The use of a three-dimensional model turns out to be essential for this application because it enables the representation of vertical stratification as well as the lateral disparities that exist in the Loire estuary.

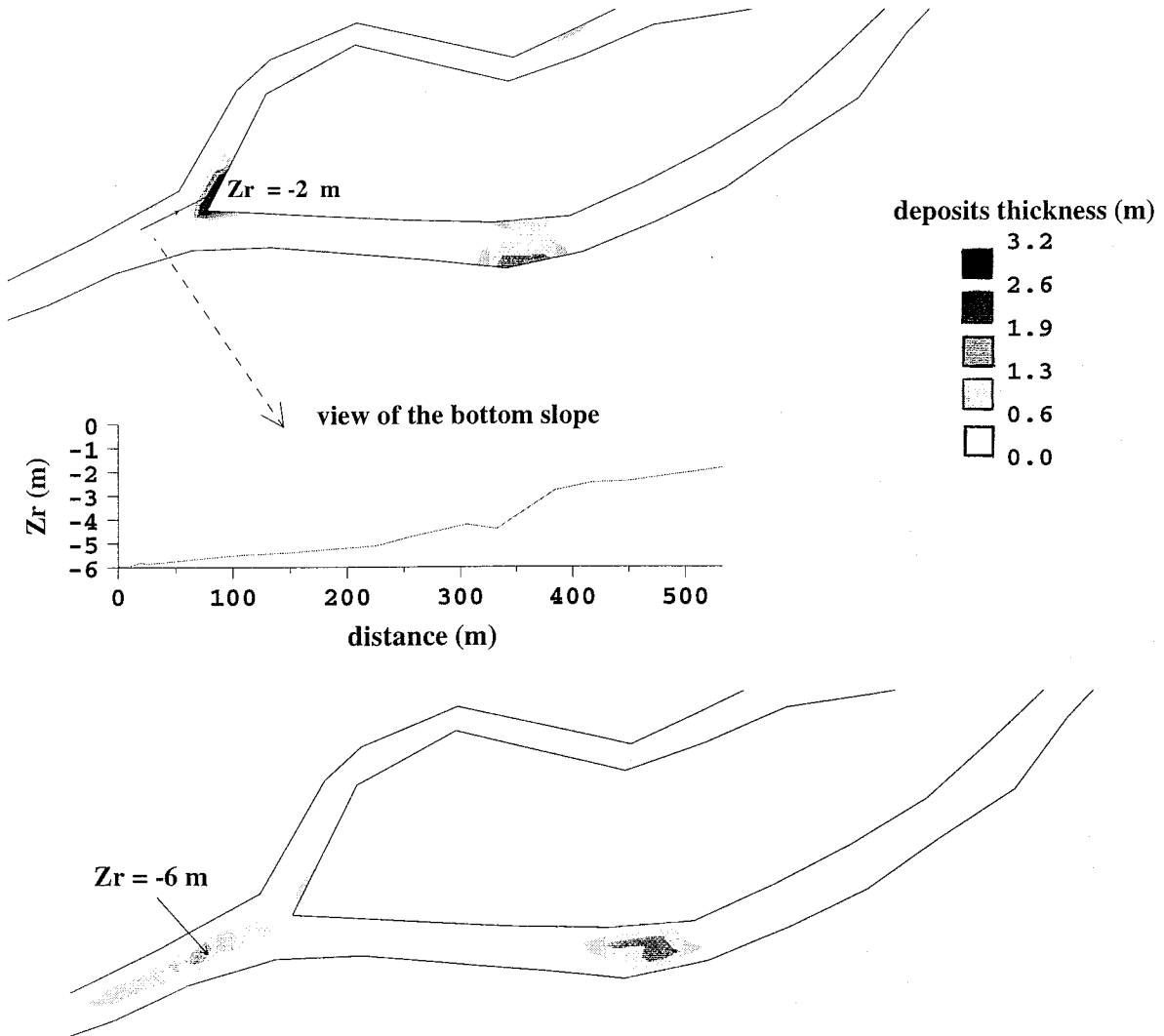


Figure 9. Influence of the fluid mud model. (a) Localization of the mud deposits near Nantes (without modelling the fluid mud movements). (b) Localization of the deposits near Nantes (with the fluid mud model)

The simulations also point out the utility of using a fluid mud model. Indeed, without modelling fluid mud movements, deposits occur at several locations in the estuary (Figure 9a) without any chance of being eroded or redistributed because currents there are very low. If fluid mud movements are represented, however, the deposited matter can slide down suitable bed slopes (Figure 9b) to the deeper parts of the estuary and can contribute to the formation of the turbidity maximum. In this model, the deposits that occur at slack water are observed mainly in the navigation channel (Figure 10), which is in agreement with field observations. For a mean river discharge, most of the deposits occur between Donges and Paimboeuf (Figure 10). This has been reported by Gallenne (1974), who carried out fluid mud measurements in the Loire estuary.

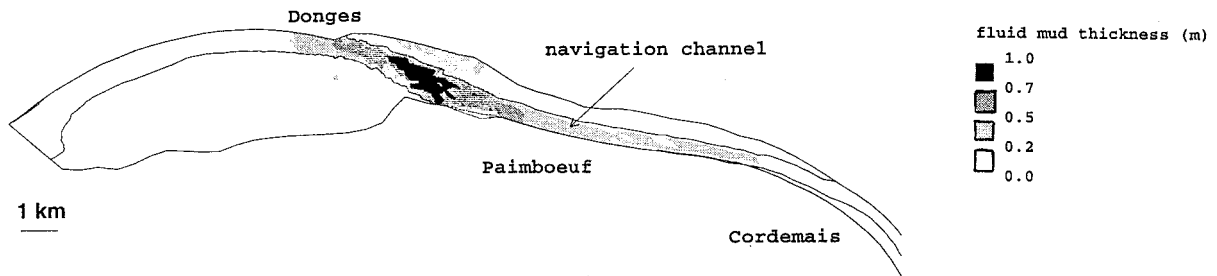


Figure 10. Spatial distribution of fluid mud at high water slack (mean river discharge)

CONCLUSIONS

The three-dimensional model of cohesive sediment transport developed in the framework of a European research programme has shown its ability to reproduce the typical features of macrotidal estuaries (formation of a turbidity maximum and movements of fluid mud). Such a model appears to be the ideal complement to *in situ* measurements. Indeed, by integrating existing measurements for its calibration, it describes the dynamic behaviour of the estuary, and by checking parameter sensitivity, it seems to be a very useful tool to determine necessary measurements to be carried out.

ACKNOWLEDGEMENTS

This work has been funded partly by the Commission of European Communities, under MAST2 Contract CT 92-0013 and partly by the French Sea State Secretary (Centre d'Etudes Maritimes et Fluviales).

REFERENCES

- CSEEL. 1983. *Etude de la masse turbide dans l'estuaire interne de la Loire*. Report of the Comité Scientifique pour l'Environnement de l'Estuaire de la Loire, Nantes, (France).
- Gallenne B. 1974. *Les accumulations turbides de l'estuaire de la Loire. Etude de la 'crème de vase'*. Thèse de doctorat, Université de Nantes.
- Gibson RE, England GL, Hussey MJ. 1967. The theory of one-dimensional consolidation of saturated clays. *Geotechnique* **17**: 216–263.
- Janin JM, Lepeintre F, Pechon P. 1992. TELEMAC-3D: a finite element code to solve 3D free surface flow problems. *Proceedings of Computer Modelling of Seas and Coastal Regions*. Southampton, UK.
- Krone RB. 1962. *Flume studies of the transport of sediment in estuarine shoaling processes*. Technical Report, Hydraulic Engineering Laboratory, University of California: Berkeley, CA.
- Le Hir P, Thouvenin B. 1992. Modélisation mathématique simplifiée de la masse turbide dans l'estuaire de la Loire. Génie Civil-Génie Côtier, Journées nationales, 26–28 Février, Nantes, France.
- Le Normant C, Lepeintre F, Teisson C, Malcherek A, Markofsky M, Zielke W. 1993. Three dimensional modelling of estuarine processes. *MAST Days and Euromar market*. Office for Publications of the European Communities: Luxembourg; 223–233.
- Malcherek A, Markofsky M, Zielke W, Peltier E, Le Normant C, Teisson C, Cornelisse J, Molinaro P, Corti S, Grego G. 1996. *Three dimensional numerical modelling of cohesive sediment transport processes in estuarine environments*. Final Report to the EC Contract MAST2-CT92-0013.
- Migniot C. 1982. *Etude de la dynamique sédimentaire marine, fluviale et estuarienne*. Thèse de doctorat d'état es-sciences naturelles. University of Paris-South.
- Odd NVM, Cooper AJ. 1989. A two dimensional model of the movement of fluid mud in a high energy turbid estuary. *Journal of Coastal Research* **5**: 185–193.
- Partheniades E. 1965. Erosion and deposition of cohesive soils. Proceedings of the American Society of Civil Engineers, *Journal of the Hydraulics Division*, **91**(HY1): 105–139.
- Sanchez Angulo MA. 1992. *Modélisation dans un estuaire à marée. Rôle du bouchon vaseux dans la tenue des sols sous marins*. Thèse de doctorat, Université de Nantes.
- Srivinas R, Mehta AJ. 1990. Observations on estuarine fluid mud entrainment. *International Journal of Sediment Research* **5**(1): 15–22.